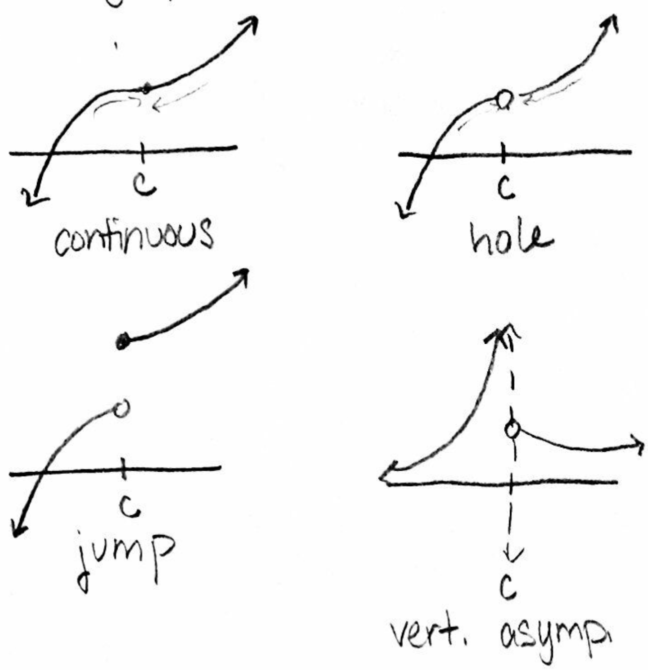
Lesson 5: Continuity

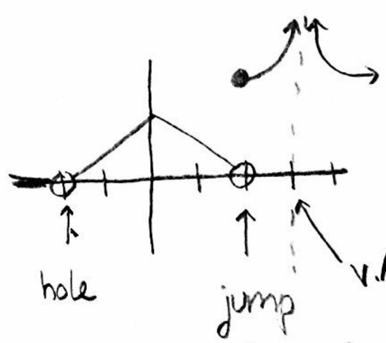
Idea: A function $f(x)$ is <u>continuous</u> at $x=c$ if you can graph it there without lifting your pencil.

Ex 1



(for rational functions, the limits from both sides is $+\infty$ or $-\infty$)

Ex 2



Def A function $f(x)$ is <u>continuous</u> at $x=c$ if $\lim_{x\to c} f(x) = f(c)$.
(Notice: this means $\lim_{x\to c} f(x)$ exists and $f(c)$ is defined.)

If $f(x)$ is continuous everywhere <u>on its domain</u>, it is a <u>continuous function</u>. (All functions from weds. are cont.)

Ex 3
$$f(x) = \frac{x^2-2x}{x^2+x-6} = \frac{x(x-2)}{(x+3)(x-2)}$$

$x=2$: hole (can cancel all $x-2$ from denominator)

$x=-3$: VA (can't cancel $x+3$ from denom, so have $\frac{a}{0}$)

Ex 4 $f(x) = \dfrac{(x+2)(x-1)}{(x+2)^2(x+3)}$

$x=-2$: VA (can't cancel all $x+2$ from denom)

$x=-3$: VA $\left(\frac{4}{0}\right)$

Ex 5 $g(x) = \begin{cases} x+1, & x<\pi \\ \cos x, & x \geq \pi \end{cases}$

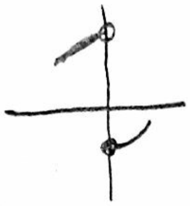
continuous

continuous

$\lim_{x\to\pi^-} g(x) = \lim_{x\to\pi^-} x+1 = \pi+1$

$\lim_{x\to\pi^+} g(x) = \lim_{x\to\pi^+} \cos x = \cos\pi = -1$

lim DNE, so $g(x)$ is discontinuous at $x=\pi$



jump at $x=\pi$